The Isolation Paradox and the Climate Change

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Abstract :

The perspective of climate change caused by human action, a problem that is characterised by a very long time lag between the moment our actions take place and their possible consequences on the climate, has given rise to renewed interest in the discount theory, and the legitimacy of its use in an inter-generational context. There is a dilemma in evaluating the cost of a future climate change : economic theory states the uniqueness of discount rate as a necessary condition of efficient allocation of scarce resources. It is not possible, then, to use a specific discount rate for economic analysis of climate change. If one uses usual discount rates (say, between 5% and 10%), the economic analysis seems capable to give possible future catastrophe only a very small present cost, et concludes that mitigation is not needed (to discount at 8% on 100 year period means to divide by 2200). If, on the other hand, one lowers the discount rate in general, it means that present generation should invest much more in all fields ; we should then consume less et save more for the sake of future and presumably richer generations. Both conclusions are counterintuitive.

The Isolation Paradox described by Amartya Sen and Stephen Marglin has been invoked to justify low discount rates in the analysis of long term issues like Climate Change. This re-examination shows some weaknesses of the argument, and cast some doubts on its general validity. However, the isolation paradox may be real as far as public goods are concerned.

Introduction

When A.K. Sen (1961) was examining optimisation of the savings rate - a problem that the Cambridge scholars Ramsey and Pigou focused on in the 20s but which was never resolved - he endeavoured to give a rigorous description of the 'isolation paradox.' In the first part of his article he reviewed some of the traditional methods used for intertemporal assessments, showing in particular the difficulty of choosing a utility function that could be maximised by economic policy, and then discussed the concept of pure time preference. While he agrees that an individual with a limited time horizon is more or less right to have a pure time preference, he considers that one cannot justify using this to determine an optimum social savings rate. He also criticises the article by Otto Eckstein (1957) which states that *"a social welfare function based on consumers' sovereignty must accept people's tastes including their intertemporal preferences"*. This is because for Sen, not all the consumers affected by this are present today.

Nonetheless, Otto Eckstein (1958) himself was one of the first to formulate what Sen called the isolation paradox: *"It is not logically inconsistent for the same person to be willing to borrow at high interest rates to increase his present consumption while voting to spend tax money to build a project from which future generations will benefit, for in the case of a vote to tax, he can be sure that the other individuals in society will be compelled to act similarly."*

However, W.J. Baumol (1952) before him expressed a similar argument but in less strong terms, suggesting that *"neither private interest nor altruism (except if he has grounds for assurance that others, too, will act in a manner designed to promote the future welfare of the community) can rationally lead him [the individual] to invest for the future, and particularly the far distant future, to an extent appropriate from the point of view of the community as a whole*."

Sen puts his argument very simply: an individual has to choose between a unit of consumption now, and three units in twenty years. But he knows that in twenty years he will be dead. He is concerned about future generations, but not enough to sacrifice one unit of his present consumption for three units of the generation that will be alive in twenty years. So he decides to consume the unit. But another man comes along and tells him that if he saves this consumption unit, he, the other man, will do the same. It is therefore not unreasonable for the first man to change his mind and agree to save. The ensuing gain for the future generation is a lot greater (six units), and he, the man, can bring this about simply by sacrificing one consumption unit.

This example may, however, imply that the first man is totally indifferent to the sacrifice agreed to by the second. This assumption is not necessarily true, and the isolation paradox can still apply even if we give more importance to the consumption of our contemporaries than to that of people in the future.

For instance, our man could give each consumption unit by the next generation a value of 0.3 of his own present consumption unit, and give each consumption unit of other members of his generation a value ’as high’, says Sen, as 0.7 of his own consumption unit. In the eyes of the first man, this gives a total investment of 1+ 0.7 units, for a gain of 0.3 * 6 = 1.8 units. On the other hand, if he were to act in isolation, our man would not be investing 1 in exchange for 3 for others in the future, valued at 3 * 0.3 = 0.9.

From this demonstration A.K. Sen shows the legitimacy of a collective decision as far as the savings rate is concerned. He then strives to define plausible variability between a maximum and a minimum savings rate, but says nothing about discount rate. This was to be dealt with later by Stephen A. Marglin (1963 *a*).

Sen goes along with a notion of utilitarianism which, like that of Mill, does not ignore feelings of altruism or empathy, but incorporates them into agents’ utility functions. We know that this notion is not that easy, because, in principle, it is likely to lead to ’double counts’ in economic valuations, particularly contingent valuations.

We may be able to avoid the risk here because what really happens is that analysis ignores the utility to future generations as such. This utility only enters into it because it is an extra utility to the present altruistic generation, prompted by the perspective of utility to future generations. The logical impact of this proposal is that we will never know the preferences of future generations. How can we avoid not knowing them? We can only assess the concern the present generation feels for future generations.

Lastly, without going deeply into it, Sen (1961) mentions another reason for the existence of the interdependence effect. People's future income does not only depend on their own decisions as to savings and investment, but also on those of other people. As individuals will never know what other people do they cannot rationally redistribute their wealth over time.

Marglin, 1963

Marglin (1963 a) does not restrict his study to extending Sen's argument to a rate of discount for investment levels. He generalises and formalises Sen's model. His argument is as follows:

Why, wonders Marglin, do governments demand that their citizens make everyday consumption sacrifices so as to make investments that will only produce benefits when all those who have made these sacrifices are dead? After rejecting "authoritarian" replies attributed to Pigou, and also "schizophrenic"[1] replies, he develops the argument put forward by William Baumol and Amartya Sen, by drawing up a model in which concern for future generations (and not for an individual's specific heirs) is the only incentive for saving. According to Marglin, we can assume that:

- time is divided into two periods, the present and the future;

[1] The "schizophrenic" reply is that people simply do not show the same economic preferences in their everyday actions and when they are asked to vote. Marglin does not reject the notion radically, but says it has limited practical scope because of its inherent dilemma: which of these preferences should be considered 'real' or more important?

- all contemporaries die at the end of the present period, and their places are taken by people who suddenly reach maturity at the beginning of the future;
- no present investment will bring any yield before the future;
- the same investment opportunities are open to everybody;
- all individuals have the same time preferences.

Now, a is the marginal value for the consumption of members of the next generation in relation to one's own, and β is the value for the consumption of one's contemporaries. The utility to the i^{th} individual is u_i, while c_i is one's own consumption, and c_p and c_f are for the consumption of the present and future generations. Lastly, δu and δc are for minor changes in utility and consumption.

Marginal utility to the i^{th} individual can be expressed thus[1]: $\delta u_i = \delta c_i + a\delta c_f + \beta(\delta c_p - \delta c_i)$

The marginal rate of conversion between present and future consumption, in this case the consumption of the next generation, is k. Therefore, every member of the present generation will receive, at the margin, a gross marginal utility gain of ak for the marginal unit invested for the benefit of the next generation.

Everyone is willing to invest for as long as the marginal utility of the investment is positive or zero: $\delta u_i = -1 + ak \geq 0$, in other words: $ak \geq 1$

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[1] According to Marglin, who goes along here with the assumption of minor changes, thus ignoring the diminishing marginal utility of income. Consequently, by hypothesis, $\frac{\partial u_i}{\partial c_i} = 1$, $\frac{\partial u_i}{\partial c_f} = a$ and $\frac{\partial u_i}{\partial (c_p - c_i)} = \beta$. The i^{th} individual's utility is expressed thus $u_i = u_i(c_i, c_p, c_f - c_i)$, and the utility variation, which is formally expressed as $\delta u_i = \frac{\partial u_i}{\partial c_i}\delta c_i + \frac{\partial u_i}{\partial c_f}\delta c_f + \frac{\partial u_i}{\partial (c_p - c_i)}(\delta c_p - \delta c_i)$ reduces to $\delta u_i = \delta c_i + a\delta c_f + \beta(\delta c_p - \delta c_i)$.

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Marglin's values for a and k are lower than Sen's: 0.1 (which he considers to be "altruistic indeed"), and 2 respectively. The product of these numbers is less than 1: nobody is willing to invest for future generations. Marglin then sets factor β at 0.15, for the value that individuals attribute to the consumption of other people in their generation. Everyone is pleased to see that others are investing, as long as $ak \geq \beta$.

Nobody is willing to invest but everybody would like to see others doing so. This is the isolation paradox. Yet one can break out of it because *"each would be willing to invest himself provided others did so, for in this case the psychic gain from others' investment would outweigh the loss on one's own investment"*. If there are n individuals in society, the change in everyone's utility if everyone invested one unit would be: $\delta u_i = -1 + akn - \beta(n-1)$.

In other words, everybody benefits as long as the marginal gain on the investment of n units exceeds the loss of utility on personal investment plus the psychic loss felt on account of the investment by other members of the community. With the assumed values of a, β and k, δu_i, the utility variation, becomes positive as soon as n is higher than or equal to 17. If n is a large number, an individual's personal sacrifice for collective investment becomes negligible compared with the satisfaction and the loss from investment by others. The only thing that counts then is the ratio of the loss attributed to investment by one's contemporaries ($n\beta$) to the gain now made possible for future generations (akn), in other words the ratio of β to ak.

The assumption on which the whole argument lies is that ak is higher than β. In other words, the product of the value given to the increase in a consumption unit by the next generation, a, by the factor multiplying the present consumption investment to give a future consumption of k is higher than the value given to the decrease of a consumption unit by every other member of the present generation β.

It is interesting to note at this stage that the variation in utility to any given individual would be even greater if everybody invested except him/her. His/her utility could then be expressed as follows: $\delta u_i = ak(n\text{-}1) - \beta(n\text{-}1) = -ak + akn - \beta(n\text{-}1) > -1 + akn - \beta(n\text{-}1)$ since $ak < 1$.

Marglin's formulation should therefore be expressed thus: "Everyone is willing to have a system adopted whereby everyone in the present generation is compelled to invest, for in this case the psychic gain from everybody else's investment would outweigh the loss on one's own investment."

In other words, as Amartya Sen was to mention later (1967), the isolation paradox is an illustration of the prisoner's dilemma (spread to *n* people) and not a problem of assurance. The solution to the former would presuppose implementing a constraint, and the solution to the latter would require assurance in the way other people behave. This is why Marglin's original formula, which is very much like that of Baumol (1952), is not strictly speaking correct, except when one considers that utility to individuals incorporating a certain empathy for future generations would mean that they would also give their own sacrifice a value - for instance a moral value such as self esteem - which would at least be equivalent to the absolute value of the sacrifice minus the extra utility resulting from the improvement for the future generation (1 - *ak*).

According to Marglin, the marginal private discount rate would be[1] $r = \frac{1}{a} - 1$, and the marginal social discount rate, assuming that there were equal investment by everyone, can be expressed thus, "*the ratio of the marginal utility to me of n dollars of current consumption, split equally among the members of the present generation, to the marginal utility to me of n dollars of consumption by the next generation, less unity*", in other words: $r^* = \frac{1 + (n-1)\beta}{na} - 1$

which, when *n* values are sufficiently high, becomes approximately: $r^* \cong \frac{\beta}{a} - 1$

Then all one has to do ito show that the marginal private discount rate is higher than the marginal social discount rate is to assume that individuals value the consumption of their contemporaries less than their own, in other words $\beta < 1$.

However, Marglin anticipates future criticism and goes on to extend his model to include the possibility of savings that will secure the future for the people saving (and/or their heirs), as well as that of the community as a whole. Each person, therefore, has two private rates, a traditional hedonistic rate of $\frac{1}{\gamma}-1$ where γ represents the marginal utility to an individual of his/her own consumption, and the private though altruistic rate of $\frac{1}{r}-1$. The rate governing the individual's decisions for unilateral investment or for consumption is the lower of the two.

Strangely, Marglin does not come to the conclusion that if, as is very likely, $\gamma > \mathrm{r}$ because individuals value their own consumption (and/or that of their heirs) more than they do that of the next generation, the traditional hedonistic rate is therefore the lower of the two. One cannot tell whether this rate is lower, higher or equal to the marginal social discount rate, since we only know that the latter is also lower than the private altruistic rate. However, Marglin limits the field for his conclusion. He rejects the normative value of the rate determined by the market for planning collective investment, and only treats as an assumption the idea that the marginal social rate of discount would be lower than the market rate[2].

Marglin goes on to suggest that the model can be extended further by abandoning the assumption that everybody has the same preference pattern. Each individual has his/her own rates of discount, whether they are marginal social ($\frac{\beta}{a}-1$), private hedonistic ($\frac{1}{\gamma}-1$) or private altruistic

[1] The *r* and *r** notations are later ones of Lind's (1964), and not Marglin's originals.
[2] All the same, this assumption alone is used in a later article (Marglin, 1963 *b*) on the problem of public investment when social and private time preferences differ.

$(\frac{1}{a}-1)$. The marginal rates do not only depend on the distribution of the cost of collective investment but also on the distribution of the extra future consumption that this enables. This is further complicated by the fact that the value each individual gives to the consumption of a contemporary or to that of a member of a future generation may be different for each contemporary and for each member of a future generation. Marglin, once again anticipating future criticism, notes that he *"personally would derive more satisfaction knowing that collective investment would add to the future consumption of individuals with low incomes than knowing it would add another yacht harbor for the enjoyment of a future millionnaire"*.

When concluding his argument Marglin explains that choosing an optimal growth rate entails an implicit compromise between the marginal time preference for, and the marginal productivity of, the investment. Given a particular work force and a particular unemployment rate, the growth rate presupposes that a certain percentage of the economic yield will be invested, which in turn presupposes that in giving a value of one to the present value of consumption flow generated by the last unit to be invested there is a marginal discount rate that corresponds exactly to the level of investment. By rejecting the notion whereby the interest rate, which is determined by an atomistic competitive market, should have any normative significance in planning for collective investment, Marglin comes to the following conclusion: *"If, for the level of investment that would emerge from a laissez-faire market, the marginal social rate of discount is lower than the market rate, then the impact of this result in a frictionless competitive model is that the community in its collective, political capacity properly sees to it - directly or indirectly - that investment opportunities with future returns too low to justify private exploitation without the intervention of the state are in fact undertaken."*

Tullock, 1964

In reply to Marglin, Gordon Tullock (1964) questions the credibility of the assumption that the product of the present valorisation of future consumption multiplied by the rate of conversion (*ak*) is higher than the valorisation of consumption by others (β). He shows firstly that one only needs to set *a* at $a = 0.0074$ (with *k* set at 2) for there to be no collective investment (whatever the size of the group, as expressed by *n*) to increase utility to the members of the present generation, since $\beta \geq ak$. Better, or worse, if $n \geq 485$ investment would have to be taxed to favour present consumption, because then $akn < 1 + \beta(n - 1)$. Therefore, according to Tullock, one should question the plausibility of the parameters that Marglin put forward without any real illustrative support. Because in Marglin's world, says Tullock, if individuals have a charity budget then they should give all of it either to the present generation or to the next, depending on whether β is higher or lower than *ak*.

At this stage in his argument Tullock introduces a very important point, one that still provides the basis for arguments in favour of discounting, particularly in articles by Thomas Schelling (1995). The point in question is the greatest foreseeable wealth of future generations. For most people, only those whose income is lower than the average should receive charitable donations. Consequently, anyone behaving altruistically should carefully weigh up the effect of giving to a poor person today rather than giving to a member of richer future generations. At any rate, says Tullock, "*collective savings of the type Marglin proposes clearly tax the poor to help the rich*".

Lind, 1964

Critiques of Marglin's work (1963) by Robert Lind (1964) are just as interesting. For Lind, the most obvious limitations of his argument are due to the lack of realism in the model itself. He explains that "*a generation which derives satisfaction solely from its own consumption may rationally undertake investments that will outlive it, as the titles to capital goods can be transferred to each generation in exchange for consumption goods in exactly the same way as* *private individuals complete such transactions in the market"*. To illustrate the point Lind builds up a model with overlapping generations.

To simplify matters, all the members of each generation are the same age, and the generations are spaced at thirty-year intervals. Each generation studies up to the age of thirty, works from thirty to sixty, and then enjoys retirement up until the time of death at ninety years old. All capital goods are owned collectively by each generation, and each generation derives well-defined utility arguing in favour of total consumption by each generation every year. A generation's consumption includes that of its children until the end of the study period. For the period before the young enter the world of work, a generation's consumption is determined by the previous generation, but after that the generation is free to allocate its income either to present consumption or to future consumption. It saves for the needs of future consumption by investing in capital that will generate service flow, and when it reaches retirement will transfer the capital accumulated to the next generation for a sum equal to the discounted value of future returns. The proceeds of the sale will be used to buy consumer goods for retirement. As death at ninety is certain, each generation uses up all its savings.

With this model, a generation can obtain compensation for the benefits of future generations by obtaining compensation from the next generation. For Lind, it follows that *"governments may undertake long-term investments in order to maximize the utility that the electorate derives from its consumption. Projects will be chosen which offer the highest return, regardless of their longevity. Thus, the fact that governments do undertake long-term investment projects does not show either that the government has violated the preferences of the electorate, or that the electors are schizophrenic with regard to preferences revealed in the market and at the ballot box, or that the present generation derives utility from the consumption of the future generation."*

Nonetheless, Lind does take up Marglin's model of two distinct generations but allows the first generation to live in the future and its members to invest for themselves or for their children and not only for future generations as a whole.

Introducing a γ factor for the marginal future utility to an individual of his/her own consumption (or that of his/her children), Lind notes that the two investment possibilities (for oneself and one's children or for future consumption by others in general) have two rates of discount, hedonistic and altruistic respectively : $\frac{1}{\gamma}-1$ and $\frac{1}{a}-1$, with the operational rate being the lower of the two.

With the logical assumption of $\gamma > a$, the former is the operational rate, and at equilibrium $\gamma k = 1$. As for the marginal social discount rate, this can be higher, lower or equal to the operational rate or traditional hedonistic rate, depending on whether β is higher, lower or equal to ak (in the same way as for Marglin).nConsequently, the requirement for equality between the social and private rates can be expressed thus: $\frac{\beta}{a} = k$ or $\frac{\beta}{a} = \frac{1}{\gamma}$

According to Lind, this condition is met if individuals consider that the decisions they make regarding savings are also valid for other people with whom they can be compared (in terms of income and utility), since they assess their (other people's) present and future consumption in the same was as they assess their own. As Lind considers this a reasonable assumption, he believes that if individuals are sufficiently comparable then the two rates are equal. In other words: if I have no reason to believe that in order to secure their own future or that of their children my neighbours should apply criteria that differ from my own, the isolation paradox no longer exists[1].

[1] This result is not at all trivial. Because the isolation paradox is not a problem of assurance but a prisoner's dilemma, it does not require that utility differ depending on the person.

If members of society are too dissimilar, then for some people the marginal private rate may be higher than the marginal social rate, and lower for others. The former would be better off if the community invested less, and the latter if it invested more. The level of investment determined by the market would not be optimum in Pareto's sense of the term, because, as Lind shows, the utility to each person can be increased by agreements encouraging some people to invest more and others less. Then what counts is the distribution of the investment, and not its absolute level. In this case, *"it is not possible, in general, to achieve a Pareto optimum by setting the interest rate at any level, and in this sense there is no social rate of discount. While such interdependence effects necessitate a political solution to the problem of investment, it is not, as Marglin's conclusion suggests, generally possible to effect an optimal solution simply by setting a rate of interest calculated to bring forth an 'optimal rate' of investment."*

Sen, 1967

Sen returns to the subject in 1967 by slightly complicating Marglin's model. He introduces the notion of direct heirs with a γ factor, which defines the value individuals give to their heirs' consumption, and a λ factor defining the portion of the return on individuals' savings that is passed on to their heirs, while λ -1 is the portion passed on to the other members of the future generation. The net gain *G(i)* of a marginal unit of savings for an individual, *i* , can be expressed thus: $G(i) = [\lambda \, . \, \gamma + (1 - \lambda)a] > 1 + (n-1)$

He then says: *"However, when we start with the amount of savings on which each has already made a decision (based on their atomistic calculation), and then consider the extra unit to be a tiny bit more, G cannot be positive or they would not have been in atomistic equilibrium. Making the usual assumptions about well-behaved and continuously differentiable functions, we shall indeed find that in the atomistic equilibrium, G = 0 for every individual"*, which reduces to:

(1) $[\lambda \, . \, \gamma + (1 - \lambda)a]k = 1$

From the point of view of the individual under consideration, the condition for the gain - from investment by n members of the community in the context of a social contract and of which a portion, h , will go to the individual's heirs - to be higher than the loss caused by the sacrifice of immediate consumption (from the same point of view), is expressed thus:

(2) $n.k[h.\ \gamma + (1 - h)a] > 1 + (n - 1)\beta$

If conditions (1) and (2) are met simultaneously, we enter the isolation paradox: no one is investing any more but each person hopes that everyone is.

A little further on in his argument, Sen drops the distinction between heirs and the other members of the future generation. As a result, $\lambda = 0$, and the $[\gamma + (1-\lambda)a]k = 1$ equation reduces to $ak = 1$. The condition for there to be an isolation paradox reduces to $1 > \beta$, which is independent of n. This seems reasonable to Sen as it did to Marglin.

We should, however, take a longer look at the $ak = 1$ equation. Marglin gave 0.1 and 2 as plausible values for a and k respectively, thus giving ak the value 0.2, while he presumed that $\beta = 0.15$. Tullock showed that by giving a a lower value, ak was lower than β, which changed the result completely. In 1961, Sen himself decided on $a = 0.3$ and $k = 3$, thus giving $ak = 0.9$, so that $\beta < ak < 1$. This time, however, Sen means to establish that $ak = 1$. Consequently, because almost necessarily $\beta < 1$ (as people value the consumption of contemporaries less than their own), the isolation paradox would be confirmed continuously.

And yet, can we really say that $ak = 1$? Just before giving ak the value he did, Marglin wrote that investment would continue as long as ak was higher than one in his model that did not yet include investment for use by investors themselves. Sen then rightly says that G, the marginal utility gain, "*cannot be positive*". If it were, there would no longer be any isolation paradox as investment would continue. On the other hand, he cannot presume that G is strictly equal to zero except by dismissing all possibility of investment by individuals for themselves.

Marglin, by extending his model to take this possibility into account, lost no time in presuming that ak = 0.2, while the isolation paradox required that $ak \leq 1$ and $ak > \beta$.

In this extended model, it is perfectly possible to imagine a situation in which k is not high enough to ensure that the ak factor is equal to 1, whatever the value of a - however low our value of the future generation's consumption[1].

In the theory on rate of interest, the rate is set when the marginal time preference for the present is equal to the marginal rate of return on investment. In the case of the latter, before equilibrium is reached the rate of return is higher (than equilibrium). The more the volume of investment increases, the fewer the opportunities for good investment, and the more the rate of return diminishes. When investment reaches marginal time preference for the present, it stops. But if preference for the present is very high - in a model where people never invest for themselves but only for others - it is very likely that no investment whatsoever will be considered sufficiently profitable. If a tends towards zero, k cannot, alas, tend towards infinity, thus enabling ak to always be equal to 1. In this case, and still with the same model, there is no investment. However, there would be investment if $ak > \beta$, a condition for the isolation paradox to exist, but Tullock has shown the uncertainty surrounding this assumption.

In a real economy, however, there is investment. To find any in Sen's model, one would have to extend it to allow investment for oneself. But that means one could say even less about the ak value or about the G. This is because the return on investment and the k conversion factor depend on the total flow of investment, and there is no *a priori* reason for ak to be equal to 1: it is more likely to be lower. As for the ratio of ak to β, it is indeterminate, as ak can be higher, lower or equal to β (or even all of these at the same time for different individuals, as Lind pointed out),

[1] One may assume that a and k apply to any generation. Naturally, k increases (in principle) with time if reinvestment is continuous. But one may assume that a diminishes after a lapse of time.

and there is nothing to prove that the rate of savings and of investment would be lower than the rate that the present members of society could decide through collective democratic procedures.

And yet it is on the basis of the assertion of a necessary relation between the rate of conversion of savings and the value given by the present generation to the consumption of the future generation - an assertion that is only valid in the sole context of a model without any investment for oneself - that Sen (1967) reasserts, in the face of critiques by Robert Lind and Gordon Tullock, not only the possibility of the isolation paradox, but its actual existence, and the existence of a marginal social rate of discount, which, in every case, is lower than the marginal private rate.

Sen also reproaches Lind for only discussing one hypothesis: that the total yield of people's savings will go to their direct heirs (in other words, $\lambda = 1$). This is not very realistic given what happens in the real world. In Sen's assessment, one would have to say $\lambda = 1$ to end up with $\gamma k = 1$. This led Sen to assert that Lind only showed that the isolation paradox did not exist in one particular case, a case that presupposed both the possibility of passing on everything to one's heirs, and what Sen calls a strict "balance of emotions", i.e. $\frac{\beta}{a} = \frac{1}{\gamma}$

The problem is that Lind's γ is not the same as Marglin's. For Marglin, γ represents the value individuals give to the consumption of their heirs. For Lind, γ is the factor given by individuals to their own consumption in the future and to that of their heirs. Lind says that $\gamma k = 1$ because he believes that people invest more for their own future and for that of those close to them, than for future generations in general (which does not prevent them deriving utility from the latter), and it is this that determines the rate for converting present consumption to future consumption. Consequently, there is no particular reason for the value of a to be expressed as $ak = 1$, which is a strict condition of the isolation paradox (because if *ak* >1, investment continues, and if *ak* <1, the isolation paradox only exists, in Sen's sense, if $ak > \beta$)[1].

However, it is worth following Sen's presentation of the general case where $0 < \lambda < 1$. If this is not a real reply to Lind, as he believes it is, it is nonetheless free of the assertion that $ak = 1$, which Sen had drawn from $G(i) = [\lambda \,.\, \gamma + (1 - \lambda)a].k - 1$ and from $G(i) = 0$, at the expense of another special case, $\lambda = 0$ (whereby individuals can leave nothing to their descendants). Sen says: *"Let us assume first that in the social contract to save more, my heir gets only λ part of my own savings, and nothing of other people's savings. In that case: $h = \lambda/N$.*

Then the required condition $n.k[h.\,\gamma + (1 - h)a] > 1 + (n - 1)\beta$ *reduces to:*

$[\lambda .\gamma + (N - \lambda)a] \,.\, k > 1 + (N - 1)\beta$.

In view of $[\lambda .\gamma + (1 - \lambda)a]k = 1$ *and the Lindian balanced emotions* ($\gamma = a/\beta$), *this is equivalent to* $\beta < 1$".

Here, we can but get lost in conjecture. We can, however, develop the following inequation:

$[\lambda .\gamma + (N - \lambda)a] \,.\, k > 1 + (N - 1)\beta$

in this form: $[\lambda .\gamma + (1 - \lambda)a] \,.\, k + (N - 1).\, ak > 1 + (N - 1)\,\beta$

which comes down to: $1 + (N - 1).\, ak > 1 + (N - 1)\,\beta$

and which still, finally reduces to $ak > \beta$, and not at all to $\beta < 1$, unless, of course, $ak = 1$, which is impossible if $\lambda \neq 0$.

In other words, after having recognised the non-existence of the paradox in a particular case, Sen, to our mind, fails to show its existence in a more general case.

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[1] The isolation paradox also exists if $ak < \beta$. It results in individuals investing more than they would really like to for future generations. The social rate of discount is then higher than the private.

To Tullock's argument, Sen replies that the greater wealth of future generations is reflected simultaneously and proportionally in the a and γ factors in the decentralised allocation of resources, and does not therefore modify the relative profitability of this or of the social contract. But here again it is a question of Lind's γ, the opposite of the conversion factor between present and future consumption (because $\gamma k = 1$), determined by the investments of individuals for themselves and for those close to them. There is therefore no particular reason for the greater wealth of future generations to lower γ to proportions comparable to a.

Another way of presenting the isolation paradox is to make use of the public goods theory, by regarding, like Marglin (1963 a), "*investment as a public good psychically consumed simultaneously by every member of the community*". John Broome (1992) sums up the argument:

"Saving for a future generation is partly a public good. Each person's saving is valued by others. There are two reasons for this. The first applies even if everyone cares only for her own descendants. In the nature of things, not all the benefit of my savings will be received by my own descendants. Inheritance taxes, amongst other things, spread them around. So when I save, I benefit other people's descendants, and those other people (my contemporaries) value that. The second reason applies if people have a wider concern for posterity beyond their own descendants. In that case, even the part of my savings that goes to my own descendants is directly valued by other people.

"But public goods are always undersupplied by the free market. The free rider problem means that people's individual savings will be less than the optimum level: less than the level individuals would themselves choose."

Here Broome implicitly takes the view that public goods are produced through contribution, by only likening savings and investment to the voluntary contributions made by each person to the part of the public good that is the well-being of future generations. In this context, people match the marginal cost of producing public goods with their own marginal will to pay. And yet the optimal level of public good production would come from matching the same marginal cost of production with everybody's marginal will to pay (a condition made by Bowen, Lindhal and Samuelson). The more the people the greater the difference. This sub-optimal state results from a lack of coordination, which does not at all presuppose that people have distinct preferences.

However, the theory that public goods are always undersupplied by the free market does not necessarily apply here. The "public good", which for the present generation is the well-being of future generations, is itself made up of private and public goods. The consumption of private goods by future generations depends on a great many factors, particularly investment by the present generation and successive generations in between. This investment could be long-term, as Lind illustrated with his model of overlapping generations. We should also bear in mind that many economic agents would like to leave goods to their heirs, and that most of them do not choose, and do not know, the moment they will die. In other words, producing "public good", the well-being of future generations - in so far as this well-being depends on the consumption of private goods - depends on mechanisms that have not been shown to lead necessarily to a sub-optimal state. Put another way, it is not certain that our great-grandparents would have tightened their belts more - or even simply wanted to - if they had known what our level of material wealth would be.

However, the well-being of future generations also depends on their consumption of, or access to, a certain number of public goods, including many environmental goods. And of course it seems that the theory of public goods can be applied to the production of these goods in the future as well as in the present. Better still, the future state of some of these goods can only result from continuous "production", in other words preserving the goods that exist today. On this account, one could easily show that the sub-optimal state that would result only from market action in this production would increase further if one took into consideration, not just the amount of the marginal will to pay on behalf of people alive today, but the supposed amount of marginal will to pay on behalf of everyone concerned, all those who are alive today as well as those who will be born in the future.

What is really in question in Broome's presentation, is the real nature of "good", which is seen by the present generation as the well-being of the future generation. Not that its "pureness" is being questioned. If it is a public good, then it is indisputably pure, with properties such as the fact that nobody can be excluded from using it, that it is mandatorily open for all to use, and is free of any congestion that may hamper its use. And if it is a good, then it is indisputably public. Nobody can reserve the exclusive right to use it; members of the present generation do not compete for its use. But is it really a good in the economic sense of the term? Here we realise the seriousness of the possible risk of dealing with altruistic types of utility in which consumption of private goods by some people becomes consumption of public goods for others, even if we seem to have avoided the risk of a double count because the utility to people who have not yet been born is not taken directly into account in the models discussed here.

Later arguments

Let us suppose that the isolation paradox exists. Can we compensate for the under-development it leads to by adopting a lower discount rate for the public sector that will lead to greater investment? No, say, in substance, Peter Warr and Brian Wright (1981), because this would lead to private individuals reducing the part of their own investment that is attributable to altruistic purposes. This in turn would lead to a simultaneous increase in the consumer's rate of time preference and the marginal rate of return on private investment. The latter is linked to the former by what is known as the "tax wedge"[1]. Here, Warr and Wright are paving the way for a more general argument on public goods: the public production of the goods displaces their private production, and does not therefore create an increase in their overall availability (Bergstrom, Blume and Varian, 1986).

David Newberry (1990), however, specifies that this argument is only valid up to the point where private production of these goods drops to zero: beyond that their public production does indeed increase their overall availability. If we take Lind's argument, voluntary investment for altruistic purposes is nil because the overall level of investment is determined by individuals' future consumption (and of their heirs). Consequently, one could conclude that when the isolation paradox exists, public investment for the benefit of future generations does not seem unreasonable.

Lastly, Sen himself was to write at a later date (Sen, 1982) that while the social rate of discount was lower than the private rate, on account of the isolation paradox, it could, however, be higher than the market rate. Indeed, among the benefits from investment that go to the 'other' members of the future generation (other than the investors or their descendants), notably thanks to royalties, one has to distinguish between the share resulting from taxation, which is included in the market rate, and the creation of jobs or production externalities[2], which are not included. *"If m is the market rate and e the rate of reward to others through sources other than taxation and transfer, then it is (m + e) that would be equated to the private rate of discount (...). Whenever e is positive, the market rate of interest will lie below the private rate of discount, and this opens up the possibility that the social rate of discount, even when below the private rate, may well exceed the market rate."*

The Climate Change and the Isolation Paradox

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[1] It is taxation more than risk that creates a difference between the consumers' rate of time preference and the marginal rate of return on private investment.

[2] Included here would be technical progress in general, and time limits on patents in particular.

What conclusion can be drawn from examining the literature on the isolation paradox? Sen has probably failed to prove that the paradox is a certainty, and the need for a marginal social time preference rate that is lower than the private or market rate[1]. But Marglin (and Sen) have probably not failed to prove the possibility of there being a lack of market coordination, or the existence of an interdependence effect, which could indeed lead to attributing a lower normative value to the market rate for public investment or public policy.

Consequently, by means of a "descriptive approach" to market reality (in the terms used by Arrow *et al*, 1996), there is a theoretical basis for collective deliberation and political decision as regards the discount rate, and, more generally, as regards the savings rate of a society, in other words, for supporting a "prescriptive approach". At this stage, on the other hand, there is nothing to nurture the belief that the rates to use for discount and investment would be necessarily lower (for the former) or higher (for the latter) than the market rates.

What's more, excellent proof for the first point can be provided through climate change itself. We have seen that the fight against climate change and the isolation paradox have the same logical structure: they are the extension to *n* agents of A.W. Tucker's famous prisoner's dilemma. Anyone who knows about the threat of climate change may wish to protect future generations from the threat, and to do this may be prepared to make a few immediate consumption sacrifices, which we will call 'investments' (even if, for instance, one fine morning it's a question of using one's bicycle rather than the car: this kind of sacrifice of immediate comfort gives future generations a little less climate change, and is therefore an investment).

Any reasonable person would, however, be aware that his/her small individual sacrifice would make very little difference to the overall threat from climate change. It is generally considered

[1] Not least of the isolation paradox's paradoxes is the fact that the rate that is supposed to best reflect the individual's will to save for altruistic purposes, which are after all quite private, is called the 'social rate', and the rate on the public market, the 'private rate'.

that no nation weighs heavily enough in the world total of greenhouse gas emissions to hope for a direct return on its own mitigation initiative. Each nation, in fact, hopes that the other nations will reduce their emissions, and so hopes to get out of having to make any effort itself. This state of affairs inevitably leads to a non-optimal overall result, unless a mechanism is put in place to ensure that every nation makes an effort. Then everyone will benefit from the effort made by others. This is the fundamental justification for international conventions such as the United Nations Framework Convention on Climate Change and the Kyoto Protocol.

What is true for nations is all the more so for individuals: no-one can inflict on themselves all the sacrifices needed to mitigate climate change, if it means doing so in a rational way within an ethics of "consistency" (anyone obeying Kant's categorical imperative would naturally choose to use a bicycle, even if he/she were the only one). In the same way that nations would act differently depending on whether the Convention existed or not, so, in their daily behaviour, individuals would not necessarily indicate their approval of instituting a mechanism forcing everyone to take action. The discount rate on the market would not reflect people's willingness to act on behalf of future generations. Their willingness can only be revealed through a process of deliberation and democratic decision, however imperfect. Therefore, there is a definite lack of market coordination for this particular category of investment.

Baumol (1968) has exactly the same reasoning in his study on "Investment as a Public Good: The Externalities Argument". "*Investment in the future*", he says, "*is of the character of a public good. National pride leads many of us to want a promising future for our country. Or looked at the other way, many of us have an uneasy conscience at leaving to future generations a world despoiled and deprived of its productive capacity. But as with national defense, it is impossible to provide a brilliant future for the nation to one of today's citizens without simultaneously making it available to all.*" However, Tullock's reply to Marglin provokes a positive reaction from Baumol[1]: *"(...) in our economy if past trends and current developments are any guide, a redistribution to provide more for the future may be described as a Robin Hood activity stood on its head - it takes from the poor to give to the rich. Average real per capita income a century hence is likely to be a sizeable multiple of its present value. Why should I give up part of my income to help support someone else with an income several times my own?"* Therefore, for Baumol *"there is no need to lower artificially the social rate of discount in order to increase further the prospective wealth of future generations."* Nevertheless, a general conclusion of this kind may not apply in some special cases: *"However, this does not mean that the future should in every respect be left to the mercy of the free market. There are important externalities and investments of the public goods variety which cry for special attention. Irreversibilities constitute a prime example. If we poison our soil so that never again will it be the same, if we destroy the Grand Canyon and turn it into a hydroelectric plant, we give up assets which (...) when once destroyed can never be supplied. All the wealth and resources of future generations will not suffice to restore them. Investment in the preservation of such items then seems perfectly proper, but for this purpose the appropriate instrument would appear to be a set of selective subsidies rather than a low general discount rate that encourages indiscriminately all sorts of investment programs whether or not they are relevant."*

The climate change example, and more generally potentially irreversible environmental externalities, do not mean we can conclude that there is a general lack of coordination for a general level of investment. But, as it were, by providing proof that the isolation paradox really did exist they give us certain legitimacy for a collective decision on the rate of discount to use for public policies, and more generally for what Lind (1964) called the "investment issue".

[1] Which was clearly not the case with the then more recent article by Sen (1967). Baumol discussed the distinction in it between the problem of assurance and the prisoner's dilemma, without paying attention to the reassertion, which counters Lind and Tullock, of the existence of the isolation paradox.

References

Arrow K.J., Cline W.R., Maler K.-G., Munasinghe M., Squitieri R., Stiglitz J.E. (1996) - "Intertemporal Equity, Discounting, and Economic Efficiency", in Bruce J.P., Lee H., Haites E.F. (Eds), *Climate Change 1995 - Economic and Social Dimensions of Climate Change*, Contribution of Working Group III to the Second Assessment Report of the Intergovernemental Panel on Climate Change, Cambridge University Press, UK, 125-144

Baumol W.J. (1952, 1965) - *Welfare Economics and the Theory of the State*, Harvard University Press, Cambridge, MA

Baumol W.J. (1968) - "On the social rate of discount", *American Economic Review*, **58** (4), 788-802

Baumol W.J. (1969) - "On the Social Rate of Discount: Comment on the Comments", *American Economic Review*, **59,** 930

Bergstrom T., Blume L., Varian H. (1986) - "On the private provision of public goods", *Journal of Public Economics*, **29**, 25-50

Broome J. (1992) - *Counting the cost of Global Warming*, White Horse Press, Cambridge, UK

Eckstein O. (1957) - "Investment Criteria for Economic Development and the Theory of Intertemporal Welfare Economics", *Quarterly Journal of Economics*, **71**, 56-85

Eckstein O. (1958) - *Water Resource Development*, John Hopkins University Press for Resources for the Future

Lind R.C. (1964) - "Further comments", *Quarterly Journal of Economics*, **78,** 336-345

Marglin S.A. (1963 a) - "The social rate of discount and the optimal rate of investment", *Quarterly Journal of Economics*, **77**, 95-111

Marglin S.A. (1963 b) - "The Opportunity Costs of Public Investment", *Quarterly Journal of Economics*, **77**, 274-289

Newbery D.M. (1990) - "The isolation paradox and the discount rate for benefit-cost analysis: a comment", *Quarterly Journal of Economics*, **105**, 235-238

Sen A. K. (1961) - "On optimising the rate of saving", *Economic Journal*, **71**, 479-496

Sen A.K. (1967) - "Isolation, Assurance, and the Social Rate of Discount", *Quarterly Journal of Economics*, **81**, 112-124

Sen A.K. (1982) - "Approaches to the Choice of Discount Rates for Social Benefit-Cost Analysis", in Lind R.C., Arrow K.J., Corey G.R., Dasgupta P., Sen A.K., Stauffer T., Stiglitz J.E., Stockfisch J.A., Wilson R. (eds), *Discouting for Time and Risk in Energy Policy*, John Hopkins University Press for Resources for the Future, Baltimore, MD, 325-353

Tullock G. (1964) - "The social rate of discount and the optimal rate of investment : comment", *Quarterly Journal of Economics*, **78**, 331-336

Warr P.G., Wright B.D. (1981) - "The isolation paradox and the discount rate for benefit-cost analysis", *Quarterly Journal of Economics*, **95**, 129-145

Warr P.G., Wright B.D. (1990) - "The isolation paradox, the social discount rate, and neutrality: further thoughts", *Quarterly Journal of Economics*, 105, 239-241